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Weak Non–Linear Waves in Fluid with Radiation Using Characteristic Method

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Abstract—*A study is made of the non linear wave propagation through a radiating gas medium that contains solid particles in suspension. The relaxation models are introduced to describe the temporal momentum and thermal non equilibrium interactions between gas and particles. The gray gas differential approximation is used for radiation. The effect of thermodynamical properties and that of the wave front curvature on the growth and decay behaviour of a finite amplitude gas dynamic disturbance headed by a planer, cylindrical or spherical are investigated.*

Keywords: *Radiation,, Non Linear Waves, Characteristic*

1. INTRODUCTION

Paper by Burger(1966), Rarity(1967), Coleman and Gurtin(!967), Becker(1970) and Chu(1970) deal with the analysis of the formation of plane shock waves in one dimensional unsteady flow with discontinuities resulting from the motion of a piston. In addition to the determination of the growth and decay properties at planer and non – planer wave fronts propagating through equilibrium flow regions of a reacting gas mixture, the present work also extends the result of Clarke(1977) to flows with non – planer geometries. We have considered, a one dimensional unsteady flow of a reacting gas mixture with discontinuities resulting from the motion of a plane, cylindrically symmetric or spherically symmetric piston. In the non – planer cases of the motion of the piston is taken to be outwards from the central axis or point of symmetry so that the wave is diverging rather than converging. It is noted that if the medium ahead of the wave is one of uniform equilibrium, then a compression wave steepens up into a shock after a finite time only if the initial discontinuity associated with the wave exceeds a critical value. It is found that the geometry of the wave affects the growth properties only indirectly in that the critical value of the initial discontinuity depends on the initial curvature of the wave front. The critical values of the initial discontinuities for cylindrical and spherical wave for which the respective wave never completely decay are found to be larger in magnitude than the corresponding value for plane of the wave on other hand if the medium ahead of the wave is in a state of disequilibrium then it is found that the reaction rate process in the flow leads to a rapid shock information; compared with the equilibrium case further it is found that an increase in the initial curvature of the wave front cause the shock formation time to increase but this stabilizing influence of the wave front cause the shock formation time to increase but this stabilizing influence of the wave front curvature unable to overcome the tendency of the wave surface to grow into a shock.

Let t denote the time, x the distance of axis or the centre of symmetry from a plane and u the gas velocity along the x- axis let the thermodynamics state of the medium be denoted by governing the one dimensional unsteady motion of a reacting gas mixture neglecting the various transport effect, are

$$\rho_t + u\rho_x + \frac{\rho}{(1-\epsilon)}u_x + \frac{\vartheta\rho u}{x} = 0 \qquad ....1$$

$$u_t + uu_x + \frac{1}{\rho(1-\epsilon)(1+\eta)}p_x = 0 \ldots \qquad 2$$

$$p_t + up_x + \rho(1-\epsilon)(1+\eta)a_e^2u_x + (1-\epsilon)^2\frac{\vartheta\rho u}{x} + Q(1-\epsilon)(1+\eta) = 0 \qquad 3$$

$$\epsilon_t + u\,\epsilon_x + \epsilon\,u_x = 0 \qquad 4$$

$$\text{Where } a_e^2 = \frac{a^2(1+\eta\rho)}{(1-\epsilon)^2(1+\eta)(1+\eta\vartheta\rho)} \qquad 5$$

Where the coefficient υ = 0, 1, 2 refers to the case of a plane, cylindrical and a spherical motion respectively. a_f is the frozen sound speed given by $a_f^2 = \frac{\gamma p}{\rho}$;

2. EQUATION IN CHARACTERISTIC FORM

Let us introduce characteristic parameter α and β which are function of x and t and which are such that

$$\frac{dx}{dt} = \frac{x_\alpha}{t_\alpha} = u + a_f$$

$$\text{Where } \beta(x,t\} = constant \qquad 6$$

$$\frac{dx}{dt} = \frac{x_\beta}{t_\beta} = u - a_f$$

Where $\alpha(x,t\} = constant$ 7

In term of new variable equation (1), (2) and (4) may be written into the following form.

$$a_f\left(t_\alpha p_\beta + t_\beta p_\alpha\right) + \frac{\rho}{(1-\epsilon)}\left(t_\beta u_\alpha + t_\alpha u_\beta\right) + 2\frac{\vartheta\rho u}{x} a_f t_\beta t_\alpha = 0 \qquad 8$$

$$a_f\left(t_\alpha u + t_\beta u_\alpha\right) + \frac{\rho}{(1-\epsilon)(1+\eta)}\left(t_\beta p_\alpha - t_\alpha p_\beta\right) = 0 \qquad 9$$

$$a_f\left(\epsilon_\beta\ t_\alpha + \epsilon_\alpha\ t_\beta\right) + \epsilon\left(t_\beta u_\alpha - t_\alpha u_\beta\right) = 0 \qquad 10$$

Also equation (2) and (3) with the help of (6) and (7) can be

$$p_\alpha + \rho a_f u_\alpha (1-\epsilon)(1+\eta) = -a_f^2 \frac{\vartheta\rho u}{x}(1-\epsilon)(1+\eta)t_\alpha + Q(1-\epsilon)(1+\eta)t_\alpha \qquad 11$$

$$p_\beta - \rho a_f u_\beta (1-\epsilon)(1+\eta) = -a_f^2 \frac{\vartheta\rho u}{x}(1-\epsilon)(1+\eta)t_\beta + Q(1-\epsilon)(1+\eta)t_\beta \qquad 12$$

3. CONDITION AT FROZEN WAVE HEAD

Assuming the region a head of the wave to be spatially uniform and at rest equ. (1), (3) and (4) yield

$$\rho_o\ t = constant \qquad ....13$$

$$p_o\ t = (1-\epsilon)^2 a_e^2 (1+\eta)\frac{\vartheta\rho u}{x} \qquad ....14$$

$$\epsilon_o\ t = 0 \qquad ....15$$

Evaluating (8), (9) and (10) at $\beta = O^+$ and using (i3) – (15)

$$a_{f0}\rho_\beta^+ = \frac{\rho}{(1-\epsilon)} u_\beta^+ \qquad ....16$$

$$t_{o\alpha} p_\beta^+ - p_{o\alpha}\ t_\beta^+ = \rho_o a_{f0}(1-\epsilon)(1+\eta) t_{0\alpha} u_\beta^+ \qquad ....$$

$$a_{f0}\left(\epsilon_\beta^+\ t_{o\alpha}\right) + \epsilon_o\ \left(t_\beta^+ u_{o\alpha} - t_{o\alpha} u_\beta^+ = 0 \qquad 18$$

Differentiating equation (11) w.r.t β and (12) w.r.t α and then subtraching one from the other and evaluating the resulting equation at $\beta = O^+$ making use 5 – 7 and 16 -18 we obtain

$$\frac{\partial}{\partial\alpha}\left[\log\left((\rho_o a_{f0})^{\frac{1}{2}} u_\beta^+\right)\right] = \left[\cap_1 - a_{fo}^2 \frac{v}{x} u_\beta^+ \rho_o\ \right] t_{o\alpha} \qquad 19$$

$$\cap_1 = -a_{fo}^2 \frac{v}{x} u_\beta^+ + Q_\beta^+ (1-\epsilon)(1+\eta)$$

Integrating yield

$$u_\beta^+ = \left(\frac{\rho_{oi} a_{foi}}{\rho_o a_{fo}}\right)^{\frac{1}{2}} u_\beta^+ \exp\int_{t_i}^{t}\left(\cap_1 - \frac{\vartheta\rho_o}{x} a_{fo}^2 u_\beta^+\right) dt \qquad (20)$$

Now diff. equation (6) w.r.t β and (7) w.r.t α and subtracting 6 from 5 and mapping use of (13) ,(14) and (15) the resulting equ when evaluating at $\beta = 0^+$ yield

$$(t_\beta^+)_\alpha + \left(\frac{\gamma(1-\epsilon)^2(1+\eta)+1}{4}\right)\frac{u_\beta^+ t_{o\alpha}}{a_{fo}} - \frac{\gamma Q_0 (1-\epsilon)(1+\eta)}{2_{f0}^{a^2}\rho_0} t_{o\alpha} t_\beta^+ = 0 \qquad 21$$

Equation 21 can be written as

$$(t_\beta^+)_\alpha + \left(\frac{\gamma(1-\epsilon)^2(1+\eta)+1}{4}\right)\frac{u_\beta^+ t_{o\alpha}}{a_{fo}} - \Lambda_2\, t_{o\alpha} t_\beta^+ = 0$$

Where $\Lambda_2 = \frac{\gamma Q_0 (1-\epsilon)(1+\eta)}{2_{f0}^{a^2}\rho_0}$

Hence, for $\in = 0$, equation reduces to the form as obtained by Clarke, and, therefore, all his conclusions follow immediately.

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